The first term on the right-hand side of (55) equals zero. Combining (53) and (55), for the integral of (50) we obtain

$$I_1 + I_2 = \int_0^{2\pi} d\xi \int_0^{2\pi} d\eta \, \frac{\partial \overline{M}_1}{\partial \eta} \cdot \overline{H}_1{}^{\perp}$$

$$= \frac{2\pi j}{\gamma \overline{M}_0{}^2} \int_0^{2\pi} d\xi \left[\omega_p \overline{M}_0 \cdot \frac{\partial}{\partial \xi} (\overline{m}_- \times \overline{m}_+) - \gamma(\overline{M}_0 \times \overline{H}_0) \cdot (\overline{m}_- \times \overline{m}_+) \right]. \quad (56)$$

However, if we use (38), we find that the integrand of (56) reduces to a total differential

$$\omega_p \overline{M}_0 \cdot \frac{\partial}{\partial \xi} (\overline{m}_- \times \overline{m}_+) - \gamma(\overline{M}_0 \times \overline{H}_0) \cdot (\overline{m}_- \times \overline{m}_+) = \omega_p \frac{\partial}{\partial \xi} (\overline{M}_0 \cdot \overline{m}_- \times \overline{m}_+).$$

The integral over one period in ξ of a total derivative with respect to ξ is zero. Thus, we have proved the correctness of (49). Finally, we have to prove (48). First, we note that $\overline{H}_1{}^{\parallel}$ is parallel to $\overline{M}_0$. Thus it is possible to write $\overline{H}_1{}^{\parallel}$ in the form

$$\overline{H}_1{}^{\parallel} = \overline{M}_0(\xi) f(\xi, \eta), \quad (39)$$

where $f(\xi, \eta)$ is an arbitrary scalar function of ξ and η. Thus,

$$\frac{\partial}{\partial \eta} (\overline{M}_1) \cdot \overline{H}_1{}^{\parallel} = \frac{\partial}{\partial \eta} (\overline{M}_1 \cdot \overline{M}_0) f(\xi, \eta). \quad (57)$$

However, from (32) and (28), we have

$$\dot{\overline{M}}_1 \cdot \overline{M}_0 + \overline{M}_1 \cdot \dot{\overline{M}}_0 = -\gamma(\overline{M}_1 \times \overline{H}_0 + \overline{M}_0 \times \overline{H}_1) \cdot \overline{M}_0 - \gamma(\overline{M}_0 \times \overline{H}_0) \cdot \overline{M}_1 = 0. \quad (58)$$

Thus, for every "process-line" originating at the source of the ξ-η plane, we have

$$\omega_p \frac{\partial}{\partial \xi} (\overline{M}_1 \cdot \overline{M}_0) + \omega_s \frac{\partial}{\partial \eta} (\overline{M}_1 \cdot \overline{M}_0) = 0. \quad (59)$$

Therefore $\overline{M}_1 \cdot \overline{M}_0$ is constant along every process line. At the origin, all process lines have the same value of $\overline{M}_1 \cdot \overline{M}_0$. Hence, $\overline{M}_1 \cdot \overline{M}_0$ is constant throughout the entire ξ-η plane and

$$\frac{\partial(\overline{M}_1 \cdot \overline{M}_0)}{\partial \eta} = 0.$$

Accordingly, (57) equals zero. This proves the correctness of (48).

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One Aspect of Minimum Noise Figure Microwave Mixer Design*

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Summary—A theory is derived which enables a direct measurement of the optimum RF impedance for minimum noise figure. This is achieved by an extension of Pound's method for loss measurements. Also, an analysis is made of the relation between minimum noise figure and maximum gain of the mixer represented as a two-port network.

The procedure consists of first matching the RF signal input terminals with short-circuited IF terminals. Next open-circuited IF terminal conditions are obtained by a circuit used by Pound. Then a reference plane is determined coinciding by preference with the plane of a maximum in the standing wave pattern of VSWR $= r$. A discontinuity is finally introduced that would have a VSWR of $\rho = \sqrt{r}$ and have its maximum or minimum at the plane of reference.

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Introduction

Microwave mixer performance has been treated in the literature [1]–[3]. In this paper the mixer is represented by a two-port network. It is assumed that the network has been optimized on an image-frequency termination basis. The aspect treated is that of determining the optimum mismatch value for the RF signal source. This is done by an extension of Pound's analysis of loss measurement. Minimum loss conditions are first determined. The relation between minimum loss and minimum noise figure is then taken up. Finally an outline is given of the determination of optimum mismatch.

Minimum Loss Requirement

Mixer behavior can be represented by a three-terminal pair network. The terminals are the signal input, IF output, and image frequency load. This representation has been confirmed experimentally and is considered adequate for practical purposes. The same nomenclature as with some treatments made elsewhere [4]–[5] will be used throughout.

It can be shown that the expressions [5] for signal source admittance giving minimum loss correspond to the conjugate-image impedances as developed by Roberts [6], and hold for any two-terminal-pair network.

It is advantageous to choose RF and IF terminals so that (111) and (112) in Torrey and Whitmer [5] can be considerably simplified. For this the following conditions must be fulfilled.

1) With the IF terminals short-circuited, the RF input is matched to satisfy

$$Y_{sc} = Y_\alpha = G_{sc} = G_{\alpha\alpha}.$$

2) The capacitor of the IF resonant circuit in Pound's apparatus [7] is adjusted to set up maximum signal reflection. This is equivalent to an addition of a susceptance, b_β to the IF terminals, which resonates out the imaginary part of $Y_{\beta\beta}$. This corresponds to open-circuited IF terminals.

3) With a standing-wave pattern resulting from condition 2 a choice of RF terminals is made so that $Y_\alpha = Y_{oc}$ is real and equal to G_{oc}.

These terminals would lie in a plane of a maximum or a minimum in the standing-wave pattern.

It may be shown that with this choice of RF terminals, $(Y_{\alpha\beta} Y_{\beta\alpha})$ is real and equal to $G_{\alpha\beta\beta\alpha}$.

Further, the loss

$$L = \frac{|Y_{\alpha\beta}|}{|Y_{\beta\alpha}|} \cdot \frac{1 + \sqrt{\dfrac{G_{oc}}{G_{sc}}}}{1 - \sqrt{\dfrac{G_{oc}}{G_{sc}}}}. \tag{1}$$

This is the underlying theory of Pound's apparatus for loss measurements.

We now write the value of (111) and (112) of Torrey and Whitmer [5], and (14) of Pound [9] under fulfillment of conditions 1–3. These equations become respectively

$$G_\alpha = G_{\alpha\alpha}\left\{1 - \frac{G_{\alpha\beta\beta\alpha}}{G_{\alpha\alpha}G_{\beta\beta}}\right\}^{1/2} \tag{2}$$

$$G_\beta = \left[G_{\beta\beta}^2 - \frac{G_{\alpha\beta\beta\alpha}G_{\beta\beta}}{G_{\alpha\alpha}}\right]^{1/2} \tag{3}$$

$$Y_\alpha = G_{\alpha\alpha} - \frac{G_{\alpha\beta\beta\alpha}}{y_\beta + G_{\beta\beta}}. \tag{4}$$

If (3) is put into (4) for y_β, the resulting equation becomes equal to (2).

Furthermore, since

$$G_{sc} = G_{\alpha\alpha} = G_o$$

where G_o is the characteristic conductance of the guide,

$$G_{oc} = G_{\alpha\alpha} - \frac{G_{\alpha\beta\beta\alpha}}{G_{\beta\beta}} = G_{sc} - \frac{G_{\alpha\beta\beta\alpha}}{G_{\beta\beta}} \tag{5}$$

$$\sqrt{G_{sc}G_{oc}} = G_{\alpha\alpha}\left[1 - \frac{G_{\alpha\beta\beta\alpha}}{G_{\alpha\alpha}G_{\beta\beta}}\right]^{1/2} \tag{6}$$

which is identical to (2).

Since the network is purely resistive with these proper terminals this result means that (6) is an image impedance of the network looking into the RF side. Thus if an RF signal source of impedance equal to (6) were connected to the network, it would correspond to minimum loss (or maximum gain). For this to be achieved it is seen from (5) that

$$G_{oc} < G_{sc}$$

and $G_{sc}/G_{oc} = r$, where r is the VSWR as measured under condition 2. It follows that G_{oc} corresponds to a plane of maximum in the standing wave pattern.

It is possible to set up an impedance $\sqrt{G_{sc}G_{oc}}$ at the plane corresponding to G_{oc}.

The VSWR ρ corresponding to this impedance is given by

$$\rho = \frac{G_{sc}}{\sqrt{G_{sc}\cdot G_{oc}}} = \sqrt{\frac{G_{sc}}{G_{oc}}} = \sqrt{r}.$$

Thus, in order to satisfy the condition of minimum loss a real impedance of magnitude $\rho = \sqrt{r}$ has to be set up at the plane of G_{oc}. This can be readily done since ρ and the plane of $\sqrt{G_{sc}G_{oc}}$ are given.

Minimum Noise Figure Condition

In an unpublished paper, Haus and Adler [8] have studied the noise behavior of a network whose internal noise contribution can be considered thermal of temperature T.

A relation

$$\frac{F - 1}{1 - \dfrac{1}{G}} = -\frac{T}{T_o}$$

is then found where F is the noise figure of the network, G is the gain of the network, and T_o is the absolute reference temperature, *i.e.*, 290°K.

An analysis of F as a function of G for the case of a passive network shows that F is a monotone decreasing function of G. Therefore it follows that the condition of maximum gain would correspond to that of minimum noise figure of the network. Consequently, if the mixer can be considered as a network whose noise contribution is of thermal nature, one would be interested in minimum loss for minimum noise figure.

Signal-Source Impedance Setup for Minimum Noise Figure

On the basis of the foregoing analysis an outline is given of the experimental setup for minimum noise figure source impedance.

1) General precautions should be taken as outlined by Wheeler and Dettinger [3].
2) The IF output is terminated by Pound's IF circuit [7]. With the switch in the short-circuit position, the VSWR is measured on the RF side and matched.
3) With the switch in the open-circuited position, tune the circuit to provide maximum VSWR $=r$ on the RF side which was previously matched, and measure r.
4) The plane corresponding to a maximum or minimum in the standing wave whose VSWR $=r$ is recorded and ρ is calculated.
5) A discontinuity is introduced in the line to set up a real impedance of magnitude $\rho=\sqrt{r}$ at the recorded plane.

Circumstances did not permit supplementing these considerations with experimental data.

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A Broad-Band High-Power Vacuum Window for X Band*

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Summary—Recent developments in high-power tubes for the 3-cm wavelength region have created a need for waveguide output windows which are capable of transmitting peak power in excess of 1 megw and average power in the neighborhood of 1 kw, and which have frequency bandwidths of about 15 per cent. This paper describes a structure which is designed to meet these electrical requirements, and which also has desirable physical and fabricational properties. A dielectric plug, which forms the vacuum seal, is used as one element of a three-element filter. The design procedure and experimental results are discussed.

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Introduction

Vacuum windows present one of the major problems at present in the design of practical high-power microwave tubes. In the 3-cm wavelength region, recent developments[1] in high-power tubes have created a need for output windows which are capable of transmitting peak power in excess of 1 megw and average power in the neighborhood of 1 kw, and which have frequency bandwidths of about 15 per cent. Such windows are used in the output waveguides of high-power tubes to allow transmission of RF power from the

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